Chem 542 Problem Set 3

1. Transformation of variables between cartesian to cylindrical coordinates means:

$$\frac{\partial}{\partial x} = \cos(\phi)\frac{\partial}{\partial r} - \frac{\sin(\phi)}{r}\frac{\partial}{\partial \phi}$$

and

$$\frac{\partial}{\partial y} = \sin(\phi)\frac{\partial}{\partial r} + \frac{\cos(\phi)}{r}\frac{\partial}{\partial \phi}$$

Now we are going to figure out what $\frac{\partial^2}{\partial x^2}$ and $\frac{\partial^2}{\partial y^2}$ are. First, determine $\frac{\partial^2}{\partial x^2}$ via the FOIL method:

(A - B) · (A' - B')

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial}{\partial x} = \left(\cos(\phi)\frac{\partial}{\partial r} - \frac{\sin(\phi)}{r}\frac{\partial}{\partial \phi}\right)\cdot\left(\cos(\phi)\frac{\partial}{\partial r} - \frac{\sin(\phi)}{r}\frac{\partial}{\partial \phi}\right)$$

Now you just must be careful that you do the "FOIL" method correctly; here it is:

AA' + AB' + BA' + BB'

$$\frac{\partial^2}{\partial x^2} = \cos(\phi)\frac{\partial}{\partial r}\cdot\cos(\phi)\frac{\partial}{\partial r} + \cos(\phi)\frac{\partial}{\partial r}\cdot\left(\frac{-\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi} + \left(\frac{-\sin(\phi)}{r}\right)\cdot\frac{\partial}{\partial \phi}\cos(\phi)\frac{\partial}{\partial r} + \left(\frac{-\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi}\cdot\left(\frac{-\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi}$$

Now we are going to now simplify it.

a. Why **can** you simplify the **AA'** term as:

$$\cos(\phi)\frac{\partial}{\partial r}\cos(\phi)\frac{\partial}{\partial r} = \cos^2(\phi)\frac{\partial^2}{\partial r^2}$$

b. Why **can't** you simplify the **AB'** term as:

$$\cos(\phi)\frac{\partial}{\partial r}\left(\frac{-\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi} = \left(\frac{-\cos(\phi)\sin(\phi)}{r}\right)\frac{\partial}{\partial r}\frac{\partial}{\partial \phi}$$

but you **can** as:

$$\cos(\phi)\frac{\partial}{\partial r}\left(\frac{-\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi} = -\cos(\phi)\sin(\phi)\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\frac{\partial}{\partial \phi}$$

c. Why **can't** you simplify the **BA'** term as:

$$-\left(\frac{\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi}\cos(\phi)\frac{\partial}{\partial r} = \left(\frac{-\sin(\phi)\cos(\phi)}{r}\right)\frac{\partial}{\partial \phi}\frac{\partial}{\partial r}$$

d. Why **can't** you simplify the **BB'** term as:

$$\left(\frac{-\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi}\left(\frac{-\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi} = \frac{\sin^2(\phi)}{r^2}\frac{\partial^2}{\partial \phi^2}$$

but you **can** as:

$$\left(\frac{-\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi}\left(\frac{-\sin(\phi)}{r}\right)\frac{\partial}{\partial \phi} = \left(\frac{\sin(\phi)}{r^2}\right)\frac{\partial}{\partial \phi}\sin(\phi)\frac{\partial}{\partial \phi}$$

Answers:

a. Because $\frac{\partial}{\partial r}$ doesn't operate on $\cos(\phi)$, so it "slips" past the derivative.

b. Because $\frac{1}{r}$ cannot slip past the $\frac{\partial}{\partial r}$ operator, but $\sin(\phi)$ can.

c. Because $\cos(\phi)$ cannot pass through the $\frac{\partial}{\partial \phi}$ operator.

d. Because $\sin(\phi)$ cannot pass through the $\frac{\partial}{\partial \phi}$ operator, but $\frac{1}{r}$ can.

2. Using the "rules" from question 1, can you determine what $\frac{\partial^2}{\partial x^2}$ is in cylindrical coordinates?

Answer:

$$\cos^2(\phi)\frac{\partial^2}{\partial r^2} - \cos(\phi)\sin(\phi)\frac{\partial}{\partial r}\cdot\left(\frac{1}{r}\right)\frac{\partial}{\partial \phi} - \left(\frac{\sin(\phi)}{r}\right)\cdot\frac{\partial}{\partial \phi}\cos(\phi)\frac{\partial}{\partial r} + \left(\frac{\sin(\phi)}{r^2}\right)\frac{\partial}{\partial \phi}\cdot\sin(\phi)\frac{\partial}{\partial \phi}$$

3. Using the "rules" from question 1, can you determine what $\frac{\partial^2}{\partial y^2}$ is in cylindrical coordinates? You must show your work on this one.

Answer: Starting with:

$$\frac{\partial^2}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial}{\partial y} = \left(\sin(\phi)\frac{\partial}{\partial r} + \frac{\cos(\phi)}{r}\frac{\partial}{\partial \phi}\right)\cdot\left(\sin(\phi)\frac{\partial}{\partial r} + \frac{\cos(\phi)}{r}\frac{\partial}{\partial \phi}\right)$$

This leads to:

$$\frac{\partial^2}{\partial y^2} = \sin(\phi)\frac{\partial}{\partial r}\sin(\phi)\frac{\partial}{\partial r} + \sin(\phi)\frac{\partial}{\partial r}\frac{\cos(\phi)}{r}\frac{\partial}{\partial \phi} + \frac{\cos(\phi)}{r}\frac{\partial}{\partial \phi}\sin(\phi)\frac{\partial}{\partial r} + \frac{\cos(\phi)}{r}\frac{\partial}{\partial \phi}\frac{\cos(\phi)}{r}\frac{\partial}{\partial \phi} =$$

$$\frac{\partial^2}{\partial y^2} = \sin^2(\phi)\frac{\partial^2}{\partial r^2} + \sin(\phi)\cos(\phi)\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\frac{\partial}{\partial \phi} + \left(\frac{\cos(\phi)}{r}\right)\frac{\partial}{\partial \phi}\sin(\phi)\frac{\partial}{\partial r} + \left(\frac{\cos(\phi)}{r^2}\right)\frac{\partial}{\partial \phi}\cos(\phi)\frac{\partial}{\partial \phi}$$

My only question to you is, did you notice that I gave you the answer on the next page before or after you worked the question?

It turns out that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} =$

$$\cos^2(\phi)\frac{\partial^2}{\partial r^2} - \cos(\phi)\sin(\phi)\frac{\partial}{\partial r}\cdot\left(\frac{1}{r}\right)\frac{\partial}{\partial \phi} - \left(\frac{\sin(\phi)}{r}\right)\cdot\frac{\partial}{\partial \phi}\cos(\phi)\frac{\partial}{\partial r} + \left(\frac{\sin(\phi)}{r^2}\right)\frac{\partial}{\partial \phi}\cdot\sin(\phi)\frac{\partial}{\partial \phi} +$$
$$\sin^2(\phi)\frac{\partial^2}{\partial r^2} + \sin(\phi)\cos(\phi)\frac{\partial}{\partial r}\cdot\left(\frac{1}{r}\right)\frac{\partial}{\partial \phi} + \left(\frac{\cos(\phi)}{r}\right)\cdot\frac{\partial}{\partial \phi}\sin(\phi)\frac{\partial}{\partial r} + \left(\frac{\cos(\phi)}{r^2}\right)\frac{\partial}{\partial \phi}\cdot\cos(\phi)\frac{\partial}{\partial \phi}$$

Note that two middle terms cancel:

$$-\cos(\phi)\sin(\phi)\frac{\partial}{\partial r}\cdot\left(\frac{1}{r}\right)\frac{\partial}{\partial \phi} + \sin(\phi)\cos(\phi)\frac{\partial}{\partial r}\cdot\left(\frac{1}{r}\right)\frac{\partial}{\partial \phi} = 0$$

Which leaves: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} =$

$$\cos^2(\phi)\frac{\partial^2}{\partial r^2} - \left(\frac{\sin(\phi)}{r}\right)\cdot\frac{\partial}{\partial \phi}\cos(\phi)\frac{\partial}{\partial r} + \left(\frac{\sin(\phi)}{r^2}\right)\frac{\partial}{\partial \phi}\cdot\sin(\phi)\frac{\partial}{\partial \phi} +$$

$$\sin^2(\phi)\frac{\partial^2}{\partial r^2}+\left(\frac{\cos(\phi)}{r}\right)\cdot\frac{\partial}{\partial\phi}\sin(\phi)\frac{\partial}{\partial r}+\left(\frac{\cos(\phi)}{r^2}\right)\frac{\partial}{\partial\phi}\cdot\cos(\phi)\frac{\partial}{\partial\phi}$$

Now we will break the sum into different parts to simplify it. I will re-represent $\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ as:

$\cos^2(\phi)\frac{\partial^2}{\partial r^2}$	$-\left(\frac{\sin(\phi)}{r}\right)\cdot\frac{\partial}{\partial\phi}\cos(\phi)\frac{\partial}{\partial r}$	$+$	$\left(\frac{\sin(\phi)}{r^2}\right)\frac{\partial}{\partial\phi}\cdot\sin(\phi)\frac{\partial}{\partial\phi}$	$+$
$\sin^2(\phi)\frac{\partial^2}{\partial r^2}$	$+\left(\frac{\cos(\phi)}{r}\right)\cdot\frac{\partial}{\partial\phi}\sin(\phi)\frac{\partial}{\partial r}$	$+$	$\left(\frac{\cos(\phi)}{r^2}\right)\frac{\partial}{\partial\phi}\cdot\cos(\phi)\frac{\partial}{\partial\phi}$	
solve in 4a	solve in 4b,c,d		solve in 5	

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4. a. Can you simplify the sum of the two leading terms to remove the angle parts:

$$\cos^2(\phi)\frac{\partial^2}{\partial r^2}+\sin^2(\phi)\frac{\partial^2}{\partial r^2}=?$$

Hint, it's really easy- what is the simplest trig identity you know?

b. Now let's deal with these two "middle" terms. We can show that if you add them together then:

$$-\left(\frac{\sin(\phi)}{r}\right)\frac{\partial}{\partial\phi}\cos(\phi)\frac{\partial}{\partial r}+\left(\frac{\cos(\phi)}{r}\right)\frac{\partial}{\partial\phi}\sin(\phi)\frac{\partial}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}$$

How? Let's start by acting $\Psi(r)\cdot\Psi(\phi)$ on the first term above:

$$-\left(\frac{\sin(\phi)}{r}\right)\frac{\partial}{\partial\phi}\cos(\phi)\frac{\partial\Psi(r)\cdot\Psi(\phi)}{\partial r}=-\left(\frac{\sin(\phi)}{r}\right)\frac{\partial}{\partial\phi}\cos(\phi)\cdot\Psi(\phi)\frac{\partial\Psi(r)}{\partial r}$$

Can you show that the expression above can be simplified to:

$$\left(\frac{\sin^2(\phi)}{r}\right)\Psi(\phi)\frac{\partial\Psi(r)}{\partial r}-\left(\frac{\sin(\phi)\cos(\phi)}{r}\right)\frac{\partial\Psi(\phi)}{\partial\phi}\frac{\partial\Psi(r)}{\partial r}$$

c. Likewise show that the second term in pt. b reduces to:

$$\left(\frac{\cos(\phi)}{r}\right)\frac{\partial}{\partial\phi}\sin(\phi)\frac{\partial\Psi(r)\cdot\Psi(\phi)}{\partial r}$$
$$=\left(\frac{\cos^2(\phi)}{r}\right)\Psi(\phi)\frac{\partial\Psi(r)}{\partial r}+\left(\frac{\cos(\phi)\sin(\phi)}{r}\right)\frac{\partial\Psi(\phi)}{\partial\phi}\frac{\partial\Psi(r)}{\partial r}$$

d. Now can you show that:

$$\left[-\left(\frac{\sin(\phi)}{r}\right)\frac{\partial}{\partial\phi}\cos(\phi)\frac{\partial}{\partial r}+\left(\frac{\cos(\phi)}{r}\right)\frac{\partial}{\partial\phi}\sin(\phi)\frac{\partial}{\partial r}\right]\Psi(r)\cdot\Psi(\phi)=\frac{1}{r}\frac{\partial\Psi(r)\cdot\Psi(\phi)}{\partial r}$$

Hint: for pt. b&c use the product rule.

Answers: a. First just factor: $\cos^2(\phi)\frac{\partial^2}{\partial r^2}+\sin^2(\phi)\frac{\partial^2}{\partial r^2}$ as $(\cos^2(\phi)+\sin^2(\phi))\frac{\partial^2}{\partial r^2}$

and note that: $\cos^2(\phi)+\sin^2(\phi)=1$. Thus $\cos^2(\phi)\frac{\partial^2}{\partial r^2}+\sin^2(\phi)\frac{\partial^2}{\partial r^2}=\frac{\partial^2}{\partial r^2}$.

b. The first step is to operate on: $\frac{\partial}{\partial\phi}\cos(\phi)\cdot\Psi(\phi)$ which uses the product rule: $\frac{\partial}{\partial\phi}\cos(\phi)\cdot\Psi(\phi)=-\sin(\phi)\Psi(\phi)+\cos(\phi)\frac{\partial\Psi(\phi)}{\partial\phi}$. Insert this into the equation:

$-\left(\frac{\sin(\phi)}{r}\right)\left(-\sin(\phi)\Psi(\phi)+\cos(\phi)\frac{\partial\Psi(\phi)}{\partial\phi}\right)\frac{\partial\Psi(r)}{\partial r}$ which simplifies to the answer.

c. Likewise $\left(\frac{\cos(\phi)}{r}\right)\frac{\partial}{\partial\phi}\sin(\phi)\frac{\partial\Psi(r)\cdot\Psi(\phi)}{\partial r}=\left(\frac{\cos(\phi)}{r}\right)\frac{\partial}{\partial\phi}\sin(\phi)\cdot\Psi(\phi)\frac{\partial\Psi(r)}{\partial r}=$

$$\left(\frac{\cos(\phi)}{r}\right)\left(\cos(\phi)\cdot\Psi(\phi)+\sin(\phi)\cdot\frac{\partial\Psi(\phi)}{\partial\phi}\right)\frac{\partial\Psi(r)}{\partial r}=$$

$$\frac{\cos^2(\phi)}{r}\Psi(\phi)\frac{\partial\Psi(r)}{\partial r}+\frac{\cos(\phi)\sin(\phi)}{r}\frac{\partial\Psi(\phi)}{\partial\phi}\frac{\partial\Psi(r)}{\partial r}$$

d. Combining the terms in pt. b&c:

$$\left(\frac{\sin^2(\phi)}{r}\right)\Psi(\phi)\frac{\partial\Psi(r)}{\partial r}-\left(\frac{\sin(\phi)\cos(\phi)}{r}\right)\frac{\partial\Psi(\phi)}{\partial\phi}\frac{\partial\Psi(r)}{\partial r}+\left(\frac{\cos^2(\phi)}{r}\right)\Psi(\phi)\frac{\partial\Psi(r)}{\partial r}$$
$$+\left(\frac{\cos(\phi)\sin(\phi)}{r}\right)\frac{\partial\Psi(\phi)}{\partial\phi}\frac{\partial\Psi(r)}{\partial r}=$$

$$\frac{(\sin^2(\phi)+\cos^2(\phi))}{r}\Psi(\phi)\frac{\partial\Psi(r)}{\partial r}=\frac{\Psi(\phi)}{r}\frac{\partial\Psi(r)}{\partial r}$$

Therefore you can state that the operator is $\frac{1}{r}\frac{\partial}{\partial r}$

5. Last one. In question 4 we have simplified all the terms but the last two:

$$\left(\frac{\sin(\phi)}{r^2}\right)\frac{\partial}{\partial\phi}\cdot\sin(\phi)\frac{\partial}{\partial\phi}+\left(\frac{\cos(\phi)}{r^2}\right)\frac{\partial}{\partial\phi}\cdot\cos(\phi)\frac{\partial}{\partial\phi}$$

Try acting on this operator with a wavefunction$\Psi(r)\cdot\Psi(\phi)$ to show that it is equal to:

$$\Psi(r)\left(\frac{1}{r^2}\frac{\partial^2\Psi(\phi)}{\partial\phi^2}\right)$$

Hint, you're going to use the product rule again.

Answer: Well, do what it says:

$$\left(\frac{\sin(\phi)}{r^2}\right)\frac{\partial}{\partial\phi}\cdot\sin(\phi)\frac{\partial\Psi(r)\cdot\Psi(\phi)}{\partial\phi}+\left(\frac{\cos(\phi)}{r^2}\right)\frac{\partial}{\partial\phi}\cdot\cos(\phi)\frac{\partial\Psi(r)\cdot\Psi(\phi)}{\partial\phi}=$$
$$\Psi(r)\left(\frac{\sin(\phi)}{r^2}\right)\frac{\partial}{\partial\phi}\cdot\sin(\phi)\frac{\partial\Psi(\phi)}{\partial\phi}+\Psi(r)\left(\frac{\cos(\phi)}{r^2}\right)\frac{\partial}{\partial\phi}\cdot\cos(\phi)\frac{\partial\Psi(\phi)}{\partial\phi}$$

Now you have to use the product rule:

$$\Psi(r)\left(\frac{\sin(\phi)}{r^2}\right)\left(\cos(\phi)\frac{\partial\Psi(\phi)}{\partial\phi}+\sin(\phi)\frac{\partial^2\Psi(\phi)}{\partial\phi^2}\right)+\Psi(r)\left(\frac{\cos(\phi)}{r^2}\right)\left(-\sin(\phi)\frac{\partial\Psi(\phi)}{\partial\phi}+\cos(\phi)\frac{\partial^2\Psi(\phi)}{\partial\phi^2}\right)=$$
$$\Psi(r)\left(\frac{\sin(\phi)\cos(\phi)}{r^2}\frac{\partial\Psi(\phi)}{\partial\phi}+\frac{\sin^2(\phi)}{r^2}\frac{\partial^2\Psi(\phi)}{\partial\phi^2}\right)+\Psi(r)\left(\frac{-\sin(\phi)\cos(\phi)}{r^2}\frac{\partial\Psi(\phi)}{\partial\phi}+\frac{\cos^2(\phi)}{r^2}\frac{\partial^2\Psi(\phi)}{\partial\phi^2}\right)=$$
$$\Psi(r)\left(\frac{\sin^2(\phi)}{r^2}\frac{\partial^2\Psi(\phi)}{\partial\phi^2}\right)+\Psi(r)\left(\frac{\cos^2(\phi)}{r^2}\frac{\partial^2\Psi(\phi)}{\partial\phi^2}\right)=\Psi(r)\left(\frac{\sin^2(\phi)+\cos^2(\phi)}{r^2}\frac{\partial^2\Psi(\phi)}{\partial\phi^2}\right)=\Psi(r)\left(\frac{1}{r^2}\frac{\partial^2\Psi(\phi)}{\partial\phi^2}\right)$$

are the negative of eachother, so when added you get 0.

6. Calculate the 1s and 1p kinetic energy (in eV) for an electron in a 1 nm sphere. Discuss whether this is a lot of energy or not so much (perhaps compare it to the temperature necessary to reach the same thermal energy). ***Hint:*** you have to be careful about finding the zero's of a Bessel vs. spherical Bessel function.

Answer: From class, we saw that the energy was related to:

$$J_{0\ or\ 1}(k\cdot a)=0$$

where $k \cdot a$ provides 0's for the spherical Bessel functions. The 1st zero for the 1-s state is at 3.14, and for the 1-p state it is 4.49, see: https://quantummechanics.ucsd.edu/ph130a/130_notes/node226.html.

Hence, the energy of the 1-s state is:

$$k \cdot a = \sqrt{\frac{2mE}{\hbar^2}} a = 3.14$$

Using a of 1 nm, etc., we find an energy of 6.02×10^{-20} J, or 0.38 eV. Similarly, for the 1-p state we find $E = 1.23 \times 10^{-19}$ J, or 0.77 eV. This is a very large amount of energy, as it corresponds to a thermal temperature of 4400K.

7. A certain one-particle, one-dimensional system has a wave function given by

$$\psi = ae^{-ibt}e^{-bmx^2/\hbar}$$

where a and b are constants and m is the particle's mass. Given that the time-dependent Schrodinger equation is:

$$-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + \hat{V}\psi = E\psi$$

a. Please find the potential energy function V.

b. While you're at it, find E (energy) for this system.

Answer: a. Starting with:

$$-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi = -\frac{\hbar}{i}\frac{\partial}{\partial t}ae^{-ibt}e^{-bmx^2/\hbar} = \hbar b \cdot ae^{-ibt}e^{-bmx^2/\hbar}$$

Factoring out ψ gives:

$$-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi = \hbar b \cdot \psi$$

And on the other side:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}ae^{-ibt}e^{-bmx^2/\hbar} = -\frac{\hbar^2}{2m}ae^{-ibt}\left\{\frac{2bme^{-bmx^2/\hbar}}{\hbar}\left(\frac{2bmx^2}{\hbar} - 1\right)\right\}$$

Simplification gives:

$$-\frac{\hbar^2}{2m}ae^{-ibt}\left\{\frac{2bme^{-bmx^2/\hbar}}{\hbar}\left(\frac{2bmx^2}{\hbar}-1\right)\right\}$$
$$=-\frac{\hbar^2}{2m}ae^{-ibt}\left\{\frac{4b^2m^2x^2e^{-bmx^2/\hbar}}{\hbar^2}-\frac{2bme^{-bmx^2/\hbar}}{\hbar}\right\}$$

Factoring out ψ gives:

$$-2b^2mx^2\cdot\psi+\hbar b\cdot\psi$$

Now going back to the original $-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+\hat{V}\psi$ expression:

$$\hbar b\cdot\psi=-2b^2mx^2\cdot\psi+\hbar b\cdot\psi+\hat{V}\psi$$

We can simply solve for V:

$$\hbar b\cdot\psi+2b^2mx^2\cdot\psi-\hbar b\cdot\psi=\hat{V}\psi$$

Therefore $\hat{V}=2b^2mx^2$

b. Since $-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi=\hat{H}\psi=E\psi$, and that $-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi=\hbar b\cdot\psi=E\psi$, clearly the energy is $\hbar b$.

8. Let's say that the last problem was secretly about the Harmonic oscillator, for which the potential energy operator is: $\hat{V}=\frac{1}{2}k_fx^2$. Can you determine what "b" actually is in terms of the angular frequency: $\omega=\sqrt{\frac{k_f}{m}}$, and show that the energy is: $E=\frac{1}{2}\hbar\omega$?

Answer: If $\hat{V}=\frac{1}{2}k_fx^2=2b^2m=\frac{1}{2}k_f$, then $b=\frac{1}{2}\sqrt{\frac{k_f}{m}}=\frac{1}{2}\omega$, and thus $\hat{V}=2b^2mx^2=2\frac{\omega^2}{4}mx^2=\frac{1}{2}m\omega^2x^2$. Likewise the energy is $E=\frac{1}{2}\hbar\sqrt{\frac{k_f}{m}}=\frac{1}{2}\hbar\omega$.

9. Write the following complex numbers in the x+iy form.

a. $(2 + 3i)^2$ **b.** $\frac{1+3i}{1-2\,i}$

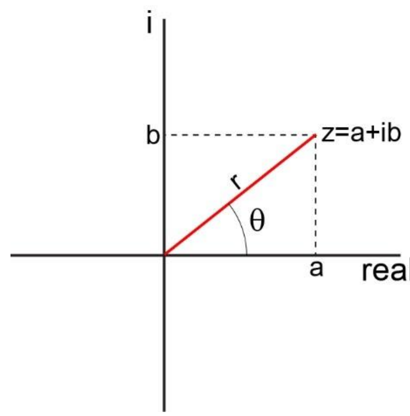
Answer: a. Just square the expression:

$$(2 + 3i) \cdot (2 + 3i) = 4 + 6i + 6i + 9i2 = 4 - 9 + 12i = -5 + 12i$$

b. Try the same approach as last time:

$$\frac{1+3i}{1-2\,i}\frac{(1+2\,i)}{(1+2\,i)} = \frac{1+2i+3i+6i^2}{1+2\,i-2i-4i^2} = \frac{-5+5i}{5} = -1+i$$

10. Although you should be aware that imaginary numbers are expressed as: a+ib, they may also come in the form: $re^{i\theta}$ where x is the real axis and y is the imaginary as shown here. Can you transform the following into that form?



a. 1+2i **b.** 1-i **c.** $\frac{1}{1+i}$.

Hint: the last one requires some additional effort.

Answer: a. Here you can see that the length and angle are defined by standard trigonometric relationships: $r = \sqrt{1^2 + 2^2} = \sqrt{5}$; $\theta = atan\left(\frac{2}{1}\right) = 1.107$. Thus, the point is $\sqrt{5} \cdot e^{1.107 \cdot \theta} = \sqrt{5} \cdot e^{i \cdot 0.35\pi \cdot \theta}$.

b. $\sqrt{2} \cdot e^{-0.7845 \cdot \theta} = \sqrt{2} \cdot e^{-i \cdot \frac{\pi}{4} \cdot \theta}$

c. $\frac{1}{(1+i)}\frac{1-i}{(1-i)} = \frac{1-i}{1-i+i-i^2} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$.

Therefore the answer is $\frac{\sqrt{2}}{2} \cdot e^{-0.7845 \cdot \theta} = \frac{\sqrt{2}}{2} \cdot e^{-i \cdot \frac{\pi}{4} \cdot \theta}$.